

A. Kovacevic
N. Stosic
I. Smith

Screw Compressors

Three Dimensional Computational
Fluid Dynamics and Solid Fluid Interaction

 Springer

Screw Compressors

A. Kovacevic N. Stosic I. Smith

Screw Compressors

Three Dimensional Computational Fluid Dynamics
and Solid Fluid Interaction

With 83 Figures

Ahmed Kovacevic
Nikola Stosic
Ian Smith

School of Engineering and Mathematical Sciences
City University
Northampton Square
London
EC1V 0HB
United Kingdom

Library of Congress Control Number: 2006930403

ISBN 3-540-36302-5 Springer-Verlag Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

Springer-Verlag is a part of Springer Science+Business Media

© Springer-Verlag Berlin Heidelberg 2007
springer.com

The use of general descriptive names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Cover-Design: Estudio Calamar, Viladasens
Typesetting by the authors and SPi

Printed on acid-free paper

SPIN: 11741589

62/3100/SPi

5 4 3 2 1 0

Preface

Screw compressors are rotary positive displacement machines, which are compact, have few moving parts and which operate at high efficiency over a wide range of speeds and pressure differences. Consequently a substantial proportion of all industrial compressors now produced are of this type.

There are few published books on the principles of their operation and how best to design them, especially in English. The authors made a first step to fill this void with an earlier work on mathematical modelling and performance calculation of these machines. This described analytical procedures which are generally adequate for most applications, especially when dealing with oil flooded machines, in which temperature changes are relatively small and thus the effect of changes of shape of the key components on performance may be neglected. This assumption permitted the use of analytical procedures based on flow through passages with dimensions that are invariant with temperature and pressure.

As manufacturing accuracy increases, clearances can be reduced and compressors thereby made smaller and more efficient. To obtain full advantage of this at the design stage, more accurate procedures are required to determine the internal fluid flow patterns, the pressure and temperature distribution and their effects on the working process. This is especially true for oil free machines, in which temperature changes are much larger and thus make thermal distortion effects more significant.

The present volume was prepared, as a sequel to the authors' earlier work and describes the most up to date results of methods, which are still being developed to meet this need. These are based on the simulation of three-dimensional fluid flow within a screw machine.

Such an approach requires the simultaneous solution of the governing equations of momentum, energy, mass, concentration and space conservation. In order to be solved, the equations are accompanied by constitutive relations for a Newtonian fluid in the form of Stokes, Fourier's and Fick's laws. Although this model is generally applicable, at least three features have been derived, without which the solution of the flow pattern within a screw machine would not be possible. Firstly, there is the application of the Euler-Lagrange method for the solution of multiphase flow of both the injected fluid and the liquid phase of the main fluid. Its equations define mass, energy and concentration source terms for the main equations. The introduction of a 'boundary domain' in which the pressure is kept constant by injection or by subtraction of mass, is the next innovation in and allows simulation of the real operational mode of a screw compressor. Finally, a rather simple method is used here for calculation of the properties of real fluids. This

permits a fast but still reasonably accurate procedure even for the calculation of processes like evaporation or condensation within the working chamber.

A computer program has been developed for Windows or UNIX oriented machines, based on the methodology, thus described. It automatically generates files which contain node, cell and region specifications as well as user subroutines for a general CFD solver of the finite volume type. The method and program have been tested on a commercial CFD solver and good simulations of positive displacement screw machines were obtained.

Four examples outline the scope of the applied mathematical model for three dimensional calculation of fluid flow and stresses in the solid parts of the screw machine. In the first example, the results for two oil-free air screw compressors with different rotor profiles are compared with each other and with results obtained from one-dimensional calculation. Advantages are found in the use of the three-dimensional simulation model, in which the suction and discharge dynamic losses are accounted for in the results.

The second example verifies the results of three-dimensional calculations with measurements obtained in an experimental test rig. The influence of turbulence upon the process in a screw compressor is also investigated. It is concluded that, despite the excessive dissipation of kinetic energy of turbulence, the overall parameters of a positive displacement machine do not significantly change if it is calculated either as turbulent or laminar flow. Investigation of the grid size influence on the accuracy of 3-D calculations is performed and it shows that coarser numerical meshes cannot capture all of the flow variations within the compressor accurately. However, the integral parameters are in all cases reasonably accurate.

In the third example the procedures for real fluid properties enabled fast calculation of ammonia compressor parameters. The importance of the oil injection port position is outlined here through the oil distribution obtained by the three dimensional calculations. Such figures of oil distribution within the screw compressor have never been found in the open literature. This achievement is therefore a step forward in understanding screw compressor internal flows.

The fourth application covers the simultaneous calculation of the solid fluid interaction. The influence of the rotor deformation on the integral screw compressor parameters caused by the change in clearance is presented together with how rotor clearances are reduced due to the enlargement of the rotors caused by temperature dilatations. This results in an increase in both, the compressor flow and power input. The influence of pressure causes the rotors to bend. For a moderate compressor pressure, the clearances gap is enlarged only slightly and hence has only a negligible influence on the delivery and power consumption. In the case of high working pressures, the rotors deform more and the decrease in the delivery and rise in specific power becomes more pronounced.

*Ahmed Kovacevic
Nikola Stosic
Ian Smith*

London,
February 2006

Notation

A	- area of the cell surface
$\mathbf{a}_1, \mathbf{b}_1$	- radius vectors of boundary points
A_1, A_2	- constants in the saturation temperature equation
b_i	- constant
B_1, B_2	- constants in the compressibility factor equation
c_i	- concentration of species
c_1-c_4	- tension spline coefficients
C_1, C_2	- coefficients of the orthogonalisation procedure
C_{drag}	- drag coefficient
C_p	- specific heat capacity at constant pressure
C, σ	- constants in $k-\varepsilon$ model of turbulence
d_i	- distance in transformed coordinate system
D_i	- mass diffusivity of the dispersed phase
d_o	- Sauter mean droplet diameter
D_1-D_4	- constants in the vapour specific heat equation
$\dot{\mathbf{D}}$	- rate of strain tensor
e_1, e_2	- cell edges maximal values in coordinate directions
$f(s)$	- adaptation variable
\mathbf{f}_b	- resultant body force
f_e	- expansion factor
f_k	- weight function
$F^r(s)$	- integrated adaptation variable
h_1-h_8	- blending functions of Hermite interpolation
h	- enthalpy
h_L	- enthalpy of evaporation
k	- turbulent kinetic energy
K_1-K_4	- coefficients for Hermite interpolation
k_p	- point counter
K	- number of points
m	- mass
m_o	- mass of species
m_i	- mass in the numerical cell
Δm_L	- mass of evaporated or condensed fluid
n	- rotor speed
Nu	- Nusselt number
p	- pressure
P	- production of turbulence energy

VIII Notation

Pr	- Prandtl number
\mathbf{q}_{ci}	- diffusion flux of species
\mathbf{q}_h	- heat flux
\mathbf{q}_k	- diffusion flux in kinetic energy equation
\mathbf{q}_ε	- diffusion flux in dissipation equation
$\mathbf{q}_{\phi S}$	- flux source in the generic transport equation
$\mathbf{q}_{\phi V}$	- volume source in the generic transport equation
\dot{Q}_{con}	- convective heat flux
\dot{Q}_{mass}	- heat flux due to phase change
\mathbf{r}	- radius vector
Re	- Reynolds number
R_i	- grid point ratio
S	- cell surface
s	- transformed coordinate
\mathbf{s}	- area vector
S_{ci}	- source term of species
S_h	- heat source term
\mathbf{S}	- viscous part of stress tensor
t	- time
T	- temperature
\mathbf{T}	- stress tensor
\mathbf{u}	- displacement vector
\mathbf{v}	- fluid velocity
\mathbf{v}_{ci}	- velocity of the dispersed phase
v_i	- Cartesian component of velocity vector
\mathbf{v}_s	- surface velocity
V	- cell volume
V_{CM}	- volume of the control mass
V_{CV}	- control volume
w	- weight factor
W	- weight function
X_ξ	- grid spacing
x, y, z	- physical coordinates
X, Y, Z	- points on physical boundaries
x_p, y_p	- coordinates of the calculation point
y_p', y_p''	- first and second derivatives in vicinity of the point P
y^+	- dimensionless distance from the wall
z	- compressibility factor

Greek symbols

α	- linear thermal expansion coefficient
α, β	- tension spline coefficients
α_1, β_1	- blending functions
δ	- Kroneker delta function
δt	- time step
ε	- dissipation of turbulent kinetic energy
ϕ	- transported property in generic transport equation
φ	- interlobe angle of male rotor
Γ_ϕ	- Diffusive term in generic transport equation
η, λ	- Lamé coefficients
κ	- thermal conductivity
μ	- viscosity
μ_t	- turbulent viscosity
π	- the ratio of a circle's circumference to its diameter
ω	- angular velocity
ρ	- density
σ	- tension spline parameter
σ_o	- oil surface tension
σ_{cv}	- normalised cell volume
ξ, η	- computational coordinates
$\hat{\xi}_0, \hat{\xi}_1$	- one-dimensional stretching functions
$\hat{\xi}$	- multi-dimensional stretching function

Subscripts

1	- male rotor
2	- female rotor
add	- injected / subtracted fluid
$const$	- constant prescribed value
D	- discharge bearings
i	- dispersed phase
L	- evaporated/condensed fluid
l	- liquid
M	- mixture
max	- maximum
min	- minimum
o	- oil
ref	- reference value
s	- grid values
S	- surface

X Notation

sat	- saturation
t	- turbulence
v	- vapour
V	- volume

Superscripts

'	- fluctuating components for time averaging
''	- fluctuating component for density-weighted averaging
k	- number of time steps

Contents

1 Introduction	1
1.1 Screw Machines.....	1
1.2 Calculation of Screw Machine Processes.....	3
1.3 Fluid Flow Calculation.....	3
2 Computational Fluid Dynamics in Screw Machines	7
2.1 Introduction.....	7
2.2 Continuum Model applied to Processes in Screw Machines.....	8
2.2.1 Governing Equations.....	9
2.2.2 Constitutive Relations.....	12
2.2.3 Multiphase Flow.....	14
2.2.4 Equation of State of Real Fluids.....	19
2.2.5 Turbulent Flow.....	22
2.2.6 Pressure Calculation.....	23
2.2.7 Boundary Conditions.....	23
2.3 Finite Volume Discretisation.....	27
2.3.1 Introduction.....	27
2.3.2 Space Discretisation.....	29
2.3.3 Time Discretisation.....	29
2.3.4 Discretisation of Equations.....	30
2.4 Solution of a Coupled System of Nonlinear Equations.....	32
2.5 Calculation of Screw Compressor Integral Parameters.....	33
3 Grid generation of Screw Machine Geometry	39
3.1 Introduction.....	39
3.1.1 Types of Grid Systems.....	42
3.1.2 Properties of a Computational Grid.....	44
3.1.3 Grid Topology.....	47
3.2 Decomposition of a Screw Machine Working Domain.....	48
3.3 Generation and Adaptation of Domain Boundaries.....	52
3.3.1 Adaptation Function.....	53
3.3.2 Adaptation Variables.....	55
3.3.3 Adaptation Based on Two Variables.....	55
3.3.4 Mapping the Outer Boundary.....	58
3.4 Algebraic Grid Generation for Complex Boundaries.....	62
3.4.1 Standard Transfinite Interpolation.....	63

3.4.2 Ortho transfinite interpolation	65
3.4.3 Simple Unidirectional Interpolation	73
3.4.4 Grid Orthogonalisation	75
3.4.5 Grid Smoothing	78
3.4.6 Moving Grid	79
3.5 Computer Program	81
4 Applications	83
4.1 Introduction	83
4.2 Flow in a Dry Screw Compressor	84
4.2.1 Grid Generation for a Dry Screw Compressor	86
4.2.2 Mathematical Model for a Dry Screw Compressor	87
4.2.3 Comparison of the Two Different Rotor Profiles	87
4.3 Flow in an Oil Injected Screw Compressor	94
4.3.1 Grid Generation for an Oil-Flooded Compressor	97
4.3.2 Mathematical Model for an Oil-Flooded Compressor	97
4.3.3 Comparison of the Numerical and Experimental results for an Oil-Flooded Compressor	98
4.3.4 Influence of Turbulence on Screw Compressor Flow	106
4.3.5 The Influence of the Mesh Size on Calculation Accuracy	114
4.4 A Refrigeration Compressor	118
4.4.1 Grid Generation for a Refrigeration Compressor	118
4.4.2 Mathematical Model of a Refrigeration Compressor	119
4.4.3 Three Dimensional Calculations for a Refrigeration Compressor	119
4.5 Fluid-Solid Interaction	123
4.5.1 Grid Generation for Fluid-Solid Interaction	123
4.5.2 Numerical Solution of the Fluid-Solid Interaction	124
4.5.3 Presentation and Discussion of the Results of Fluid-Solid Interaction	125
5 Conclusions	131
A Models of Turbulent Flow	133
B Wall Boundaries	139
C Finite Volume Discretisation	143
References	155

Introduction

1.1 Screw Machines

The operating principle of screw machines, as expanders or compressors, has been known for over 120 years. Despite this, serious efforts to produce them were not made until low cost manufacturing methods became available for accurate machining of the rotor profiles. Since then, great improvements have been made in performance prediction, rotor profile design and manufacturing techniques. Screw compressors are now highly efficient, compact, simple and reliable. Consequently, they have largely replaced reciprocating machines for the majority of industrial applications and in many refrigeration systems.

Screw compressors and expanders are positive displacement rotary machines. They consist essentially of a pair of meshing helical lobed rotors, which rotate within a fixed casing that totally encloses them, as shown in Figure 1-1.



Figure 1-1 Screw Compressor Components

Although screw machines can function as either expanders or compressors, their overwhelmingly common use is as compressors, of which there are two main types. These are oil flooded, commonly known as oil injected, and oil free compressors. An example of each, with similar rotor sizes, is shown in Figure 1-2.

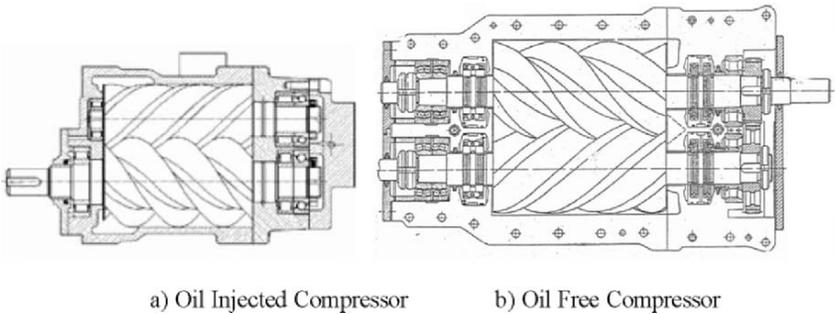


Figure 1-2 Types of Screw Compressor

In oil injected compressors, a relatively large mass, though a very small volume, of oil is admitted with the gas to be compressed. The oil acts as a lubricant between the contacting rotors, a sealant of any clearances between the rotors and between the rotors and the casing and as a coolant of the gas during the compression process. This cooling effect improves the compression efficiency and permits pressure ratios of up to approximately 15:1 in a single stage, without an excessive temperature rise, by maintaining an oil:gas mass ratio of 4:1 or even more. The effects of thermal expansion are then relatively small and now that screw compressor components can be manufactured with tolerances of the order of $\pm 5\mu\text{m}$, internal clearances can be as little as 30-60 μm .

In oil free compressors, only gas is admitted into the working chamber. External timing gear is therefore needed, in order to prevent rotor contact, and internal shaft seals have to be located between the bearings at each end of the rotor shaft and the main body of the rotor. The shaft seals are needed to prevent oil, which is supplied to the bearings through an external lubrication system, from entering the working chamber and thereby contaminating the gas being compressed. Because there is no injected oil to cool the gas in this type of machine, the temperature rise of the compressed gas is much higher than in oil flooded compressors and pressure ratios are therefore limited to approximately 3:1, depending on the type of gas being compressed. Above this value the temperature rise associated with compression creates problems related to rotor and casing distortions. Clearances therefore have to be much larger in order to avoid contact between the rotors or between the rotors and their casing caused by thermal distortion. It is believed that the adiabatic efficiency of oil free compressors could be increased by as much as 10%, if minimum safe clearances could be predicted accurately.

1.2 Calculation of Screw Machine Processes

Up till approximately 1980, screw compressors were designed assuming an ideal gas in a leak proof working chamber going through a compression process which could reasonably be approximated in terms of pressure-volume changes by the choice of a suitable value of exponent “n” in the relationship $pV^n = \text{Constant}$.

To improve on this procedure it was first necessary to obtain an algorithm which could be used to estimate the trapped volume between the rotors and the casing and the areas of all leakage passages, at any rotational angle. The latter are formed by clearances between the rotors and between the rotors and the casing. In addition, the area of the inlet or exit passage exposed to bulk flow of fluid into or out of the working chamber had to be obtained where applicable.

The assumption of dimensionless non-steady bulk fluid flow and steady one dimensional leakage flow through the working chamber, together with suitable flow coefficients through the passages and an equation of state for the working fluid, made it possible to develop a set of non-linear differential equations which describe the instantaneous flow of fluid, work and heat transfer through the system. These could be solved numerically to estimate pressure-volume changes through the suction, compression and delivery stages and hence determine the net torque, power input, fluid delivery and isentropic and volumetric efficiencies of a compressor. More details of this are given in the authors’ earlier volume *Stosic, Smith and Kovacevic*, (2005).

1.3 Fluid Flow Calculation

In recent years there has been a steady growth in the use of Computational Fluid Dynamics (CFD) as a means of calculating 3-D external and internal flow fields. It is widely used today for estimating flow in rotating machinery and specialised codes have been developed for this to allow faster calculations.

Many books on fluid dynamics such as *Bird et al* (1962), *Fox and McDonald* (1982) and *White* (1986) contain a detailed derivation of general conservation laws. Three main groups of methods have been developed through the years as described by *Ferziger and Perić* (1995). These are the finite difference, finite element and finite volume methods. It is believed that the finite difference was first described by *Euler* in the 18th century but, more recently, *Smith* (1985) gave a comprehensive account of all its aspects.

The finite element method was initially developed for structural analysis, but later has also been used for the study of fluid flow. It has been described extensively by many authors, such as *Oden* (1972), *Fletcher* (1991) and *Zienkiewicz and Taylor* (1991).

A summary of the finite volume method is given by *Versteeg and Malalasekera* (1995). Since the finite volume method has already been used to solve problems involving unsteady flow with moving boundaries and strong pressure-velocity-density coupling, it is of particular interest for this book. The ‘space conservation

law' was introduced by *Trulio and Trigger* (1961) and used in conjunction with a finite difference method. The importance of the space conservation law was discussed by *Demirdžić and Perić* (1988) and introduced to the finite volume method for prediction of fluid flow in complex domains with moving boundaries by the same authors (1990) and also by *Demirdžić, Issa and Lilek* (1990). They followed the attempts of many other authors to apply it to solve some special cases. Typically, *Gosman and Watkins* (1977), *Gosman and Johns* (1978) and *Gosman* (1984) reported the calculations of flows in a cylinder with moving boundaries. *Stošić* (1982) applied the method to internal unsteady flows of a compressible fluid. *Thomas and Lombard* (1979) presented solutions of steady and unsteady supersonic flows while *Gosman* (1984) and *Durst et al* (1985) reported that simple transformation of the conservation equations enables easy discretisation when only one of the domain boundaries moves in one direction. *Perić* (1985) introduced a finite volume methodology for prediction of three-dimensional flows in complex ducts where, among others, he gave an evaluation of various pressure-derivation algorithms for orthogonal and non-orthogonal grids. Additional analysis on pressure-velocity coupling is given by *Perić* (1990) and later discussed and improved by *Demirdžić et al* (1992) and *Demirdžić and Muzaferija* (1995) where they applied the method simultaneously to fluid flow and solid body stress analysis. Turbulence modeling is discussed by many authors, among whom *Hanjalić* (1970) gave an essential introduction to its wider use. *Bradshaw* (1994) and *Hanjalić* (1994) gave good summaries on the subject. *Lumley* (1999) outlined important subjects on turbulence in internal flows of positive displacement machines.

Despite a large number of publications on CFD, little has been written on its use for the analysis of flow through screw machines. This is mainly due to the complexity of both, the machine configuration and the flow paths through them. Some existing commercial CFD codes have facilities that can cope with the complex geometry of screw machines. Unfortunately, these codes need to be improved in order to give useful results. In addition, a pre-processor needs to be developed to generate a numerical mesh that describes their shape with sufficient accuracy and allows for the complex stretching and sliding motion associated with the flow.

With the advance of computing, it is now possible to predict internal flow in screw compressors by 3-D methods so that the internal pressure and temperature distribution can be estimated throughout the machine. Further, this can be used as a basis for determining the distribution of injected oil in oil flooded machines, as a means to estimate thermal distortion within an oil free compressor and to design inlet and exit port passages with minimum flow losses.

Such a procedure makes it possible to reduce the size of screw compressors by bringing internal leakage to a minimum. This would improve the adiabatic efficiency of such machines by virtue of the reduced internal losses and greatly reduce the cost of developing new products by cutting the time and cost of experimental testing and development.

Despite a significant increase in the number of papers published recently in the area of computational fluid dynamics, only a few deal with the application of computational fluid dynamics to screw compressors. All of them are recent papers by *Kovacevic, Stosic and Smith* published between 1999 and 2005. These papers

introduced 3-D numerical analysis to the screw compressor world. Their latter papers are related to both, grid generation in screw compressors and 3D numerical performance estimation, *Kovacevic et al* (2003) and (2005). These include fluid solid interaction in screw machines, *Kovacevic et al* (2004).

Computational Fluid Dynamics in Screw Machines

2.1 Introduction

As computer technology and its associated computational methods advance, the use of 3-D Computational Fluid Dynamics (CFD) to design and analyse positive displacement machinery working processes is gradually becoming more practicable. In general, the CFD modelling process can be split into four phases.

The first phase is concerned with defining the problem that has to be solved. Both the ease of solution and implementation of results into the design process are heavily dependent on this critical starting step. Two different approaches are available for screw machines. The first is to select one interlobe on the main rotor and the corresponding interlobe on the gate rotor in order to make a computational domain. This is probably the easiest to implement but takes no account of important phenomena such as interlobe leakage, blow-hole losses, oil injection and oil distribution. Another approach assumes that the whole domain of a screw machine is analysed. This includes the suction chamber and its port, the compression or expansion chamber with its moving rotor boundaries and the discharge system of the machine. By this means, the leakage paths and any additional inlet or outlet ports are included in the domain to be analysed. Realism in representing the machine working process gives a large advantage to this approach. The design procedure and the CFD numerical analysis can then be easily connected and interchanged and the calculation of the operational parameters of such machines is thereby facilitated. Unfortunately, such a complex geometry cannot be represented by a small number of computational points.

In the second phase, a mathematical model that is capable of describing the problem has to be selected. There are again two types of situation. The first is where an adequate mathematical description exists and can be used, e.g. heat conduction, elastic stress analysis and laminar fluid flow. The second is where such a description either does not exist or is impracticable to use, e.g. non-linear stress analysis and turbulent fluid flow. In the case of positive displacement machines, it is unlikely that any analytical solutions exist. This is because highly compressible flow appears inside both domains with turbulent flow regimes and domains with low Reynolds numbers. There is additional non-linearity introduced by two-phase flow, particle flow, moving and stretching domains and sliding boundaries. Due to

all these, the mathematical model implemented here needs to cope with a variety of different requirements. It is based on the general laws of mass, momentum, energy and space conservation. The resulting system of governing equations is not closed because it contains more unknowns than resulting equations. It is closed by constitutive relations, which give information about the response of a particular continuum material to external influences. The whole concept of mathematical modelling is based on a phenomenological approach which employs the principle of a continuum as the physical background. It can be applied only when an elementary part of material or the smallest characteristic length of the flow, which has to be analysed, is much bigger than the mean free molecular path. Fortunately, this condition is fulfilled for the majority of fluids and practically for all solid structures.

The mathematical description of problems in continuum mechanics is very rarely amenable in a closed-form of analytical solution and an iterative numerical procedure is thus the only alternative that can be applied to solve models in positive displacement machines. Numerical methods transform the differential equations of the mathematical model into a system of algebraic equations. The third phase is therefore to select the discretisation method. To do that a number of approximations are made: the continuum is replaced by a set of computational points with finite distances between them in space and time, while the continuous functions which represent the exact solution of the mathematical model are approximated by polynomials, typically of a second order. Because of the complexity of positive displacement machines, the standard approach to spatial discretisation is not applicable and a special grid generation method has to be developed and applied to them. The equations are discretised by the finite volume method, which appears to have a more conservative form of governing laws than any other numerical method. The result of discretisation is a system of algebraic equations the size of which depends on the number of numerical cells.

The resulting set of algebraic equations is then solved by approximate iterative methods. Iterations are necessary due to the non-linearity of the mathematical model. Even for linear problems, an iterative solution method is usually more efficient than a direct one. In addition, iterative solution methods are less sensitive to round-off errors due to the finite accuracy of the computer arithmetic.

2.2 Continuum Model applied to Processes in Screw Machines

A mathematical model of the transport processes, which exist within both twin screw and other types of positive displacement machine, is presented here. It includes the mass, momentum and energy conservation equations in integral form, a space conservation law, which has to be satisfied for problems with a moving mesh, constitutive relations required for the problem closure, a model of dispersed flow, models of turbulence in fluid flow and boundary conditions.

All the equations are presented in a symbolic coordinate-free notation which directly conveys the physical meaning of particular terms without unnecessary ref-

erence to any coordinate system. However, numerical solution of these equations requires a coordinate system and vectors and tensors have to be specified in terms of their components.

2.2.1 Governing Equations

Fluid contained within a screw compressor can be gas, vapour or a wet mixture of liquid and vapour. In some cases, it can be pure liquid. Its density varies with both pressure and temperature. The compressor flow is governed by equations based on the general laws of continuity, momentum and energy conservation. The most general approach is to write these equations in integral form and apply them to an arbitrary part of the fluid or solid continuum of volume V , which is bounded by a moving surface S , as shown in Figure 2-1.

Reynolds' transport theorem can be expressed as:

$$\frac{d}{dt} \int_{V_{CM}} \rho \phi dV = \frac{d}{dt} \int_{V_{CV}} \rho \phi dV + \int_S \rho \phi (\mathbf{v} - \mathbf{v}_s) \cdot d\mathbf{s} \quad (2.1)$$

where, V_{CM} is the volume of the control mass, V_{CV} is the control volume enclosed by the surface S . Vector $d\mathbf{s}$ stands for the outward pointing surface vector, defined by its unit vector \mathbf{n} and surface area dS as $d\mathbf{s} = \mathbf{n} dS$. In equation (2.1), ϕ represents any intensive property based on mass, momentum, energy, concentration or other parameter.

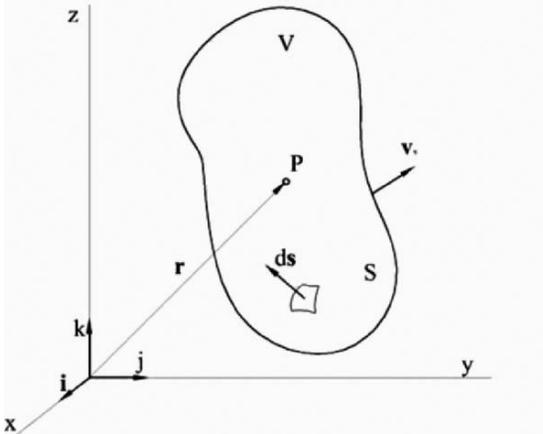


Figure 2-1 Control volume of part of the continuum

Vector \mathbf{v}_s is the velocity with which the surface of the control volume moves. If the control volume is fixed, then its surface velocity $\mathbf{v}_s = 0$. Equation (2.1) then makes the rate of change of the amount of property in the control mass equal to the sum of the rate of change of that property within the control volume and its net